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Observation on the Role of Bulk Viscosity in Present Scenario of the Evolution in FRW Model Universe

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Abstract

In this paper, we have investigated the role of bulk viscosity in present scenario of the evolution in FRW model universe in the framework of Lyra's geometry. We derived the field equations when the source for energy-momentum tensor is composed of a bulk viscous fluid with cosmic strings. The Einstein's field equations are solved by assuming a constant deceleration parameter. In this work, the displacement vector is considered to be a function of time. The kinematic and physical properties of the model are also discussed by using some acceptable physical assumptions of scale factor for flat, open, and closed universe.

Keywords: FRW, Lyra Geometry, Bulk Viscous Fluid, Cosmic Strings, Deceleration Parameter

Introduction

From the theoretical work carried out by most researchers and experimental evidences, it was proposed that our universe expanded rapidly just after big-bang within a small fraction of second. The modern findings in cosmology tell us that the universe is expanding and accelerating [1-3]. Observations from type-Ia Supernova [4-7], CMB radiation [8,9] and LSS [10] are the evidence that the current universe is having an accelerated expansion, rather than slowing down as predicted by the big bang theory [11]. Scientists are trying to solve this accelerating universe by assuming various probabilities. But till today, they could not arrive at a satisfactory conclusion on such strange behavior of the universe. The behaviour of late-time acceleration of the universe cannot be satisfactorily described by the general theory of relativity, although it is considered as the most successful theory in describing the early evolution of the universe. Cosmologists have arrived at two possible approaches to answer this cosmic accelerating expansion. One of such approaches is to introduce dark energy which dominates the universe and has associated with negative pressure. The second consideration is to modify Einstein's general theory of relativity.

Recently, some of the alternative theories of gravity are studied by many researchers. Among them, the most significant theories are the Weyl theory [12], Lyra geometry [13], Brans-Dicke theory [14], f(R) theory [15] and f(R,T) gravity [16]. Weyl's theory is a generalized theory of the Riemannian manifold to unify gravitational and electromagnetic fields. But due to the drawbacks in integrability feature, this theory could not attract many researchers. Later, Lyra removed this non-integrability feature by introducing a gauge function into Reimannian structure. Many researchers investigated Lyra geometry in four or higher-dimensional space-time. Aygun et al. [17] showed non-survival of the massive scalar field for an anisotropic Marder type universe in the framework of Lyra and Riemannian geometries. Rahaman and Bera [18] studied Kaluza-Klein cosmological model within the context of Lyra geometry in higher dimensions. Singh et al. [19] have investigated five-dimensional homogeneous cosmological models by considering bulk viscosity and variable gravitational constant in Lyra geometry. Many prominent researchers [20-23] have investigated different cosmological models within the framework of Lyra geometry and modified gravity.

The cosmological study in higher dimensions has become a great significance in investigating the early evolution of the universe. For many years, scientists are trying to unify four fundamental interactions to investigate the universe in the early epoch. In view of Kaluza-Klein theories, many authors [24-27] have studied higher dimensional cosmological model. Chodos and Detweiler [28] showed the possibility of extra dimensions of space. At the very early stage, all the dimensions (4+1) exist on the same scale. Later, during evolution, the fifth dimension shrinks and becomes unobservable. Guth [29] and Alvarez [30] in their papers presented the cosmological scenario of existing entropy on a large-scale during compactification of its extra dimension. The present space-time in 4-dimension can be modeled reverting to space-time in higher dimensions such that the universe at an early age can be thought of having more than 4-dimensions. Yilmaz [31] solved Kaluza-Klein cosmology in five dimensions for quark matter distribution of the universe attached to cloud string and domain wall in the framework of general relativity. Rahaman et al. [32] investigated higher-dimensional string theory in Lyra geometry. The strong evidence for the concept of extra dimensions has motivated some researchers [33-37] to study cosmology in multidimensional space-time geometry.

In the past and in the recent years, some researchers are showing much interest in FRW space-time geometry because of its spatial homogeneity and isotropy. At a large-scale structure, the current universe is represented by FRW models. The FRW metric is associated with the high symmetry of these backgrounds. Due to its high degree of symmetry, FRW models give a better explanation in most of the physical situations and therefore become useful in dealing with many complicated geometries. Also, in FRW cosmology, the metric is consistent with the framework of Mach's principle [38]. Beesham [39] solved FRW cosmological model using the idea of time dependent displacement vector field.

In cosmology, to investigate the physical scenario during the formation of the early universe, the concept of string theory provides a better understanding of evolution, before particles creation in the universe. Scientists believed that just after the big-bang, the universe undergoes a spontaneous symmetry breaking during the phase transition which results in a topological stable defect called cosmic strings. Cosmic strings are the main source in rising density perturbations that are responsible for galaxy formation in the early universe [40-42]. Also, the bulk viscosity mechanism in cosmology describes the present scenario of high entropy and accelerated expansion of the universe. At an early epoch, the coupling of neutrinos disappears, and matter distribution in the universe act as a bulk viscous fluid [43]. The bulk viscous fluid is associated with the transition from massive superstring models to fewer models, the occurrence of the gravitational string, and the effects of particle creation in a GUT era. Hence, the study of one-dimensional cosmic strings together with bulk viscous fluid has become an important subject in investigating cosmological models. The study of bulk viscous string cosmology in higher dimensions in Lyra manifold was started by Mohanty et al [44]. Reddy et al. [45,46] studied Kaluza-Klein cosmology with bulk viscosity and string in five dimensions in the modified theory of gravity. Vidyasagar et al. [47] have discussed a Bianchi-type universe filled with the same type of matter in Brans-Dicke theory of gravity. Naidu et al. [48,49] investigated a different class of Bianchi universe with bulk viscous cosmic string in the context of both in f(R, T) gravity and in the scalar-tensor theory of gravitation formulated by Saez and Ballester [50]. Reddy and Kiran [51] in their paper found that bulk viscous string cosmological model cannot exist in Bianchi type III space-time in f(R, T) gravity, while in general relativity this reduces to vacuum model.

Motivated by the above kinds of literature, in the present work, we investigate the role of bulk viscosity in Friedman-Robertson Walker (FRW) model universe in a higher dimension in Lyra geometry. We consider one-dimensional cosmic string along with bulk viscosity as the source for energy-momentum tensor. The paper is presented as follows. In section 2, we presented field equations by higher dimensional FRW metric in Lyra geometry. In Section 3, we solve the field equations with some acceptable physical assumptions of scale factor for flat, open, and closed models in three different cases. Kinematic and physical interpretations are given in Section 4. Section 5 has a concluding remark.

Metric and field equations

We consider Robertson-Walker metric in five dimensional space-time in the form

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + Sin^2\theta d\phi^2)\right] - S^2(t)d\Psi^2 \quad (1)$$

Where $R(t)$ is the scale factor of the universe, $k = 1, 0, -1$ for space of positive, vanishing and negative curvature representing closed, flat and open models of the universe respectively. The fifth co-ordinate Ψ is also assumed to be space like coordinate.

The Einstein field equations based on Lyra's geometry in normal gauge is given by Sen [52] as

$$G_{ij} + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_k\phi^k = -T_{ij} \quad (2)$$

$$R_{ij} - \frac{1}{2}Rg_{ij} + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_k\phi^k = -T_{ij}$$

with $\frac{8\pi G}{c^4}$. The first two terms of eqn. (1) are Einstein tensor G_{ij}, ϕ_i is the displacement vector and other symbols have their usual meaning as in Riemannian geometry. The time like displacement vector ϕ_i in Eqn (1) is given by

$$\phi_i = (0,0,0,0,\beta(t)) \quad (3)$$

The non-vanishing components of the left hand side of equation (2) for the metric (1) are given by

$$G_0^0 = \frac{3\dot{R}^2}{R^2} + \frac{3\dot{R}\dot{S}}{RS} + \frac{3k}{R^2}$$

$$G_1^1 = G_2^2 = G_3^3 = \frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{2\dot{R}\dot{S}}{RS} + \frac{\ddot{S}}{S} + \frac{k}{R^2} \quad (4)$$

$$G_4^4 = \frac{3\ddot{R}}{R} + \frac{3\dot{R}^2}{R^2} + \frac{3k}{R^2}$$

Where, an overhead dot indicates ordinary differentiation with respect to t.

The energy momentum tensor T_{ij} for cloud of massive strings with bulk viscosity is given by Landu and Lifshitz [53],

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j - \xi\theta(u_i u_j - g_{ij});$$

$$i, j = 0, 1, 2, 3, 4 \quad (5)$$

Here, ρ is the rest energy density of the cloud of strings with particles attached to them, λ is the string tension density of the strings and ξ is the co-efficient of bulk coefficient. If the particle density of the configuration is denoted by ρ_p, then we have

$$\rho = \rho_p + \lambda \quad (6)$$

The velocity u^i describes the five-velocity, which has components $(1,0,0,0,0)$ for a cloud of particles and x^i represents the direction of string that satisfies the condition

$$u^i u_i = x^i x_i = -1; \quad u^i x_i = 0 \quad (7)$$

So that we have

$$T_0^0 = \rho, \quad T_1^1 = T_2^2 = T_3^3 = \xi\theta,$$

$$T_4^4 = \xi\theta + \lambda \quad (8)$$

$$T = T_0^0 + T_1^1 + T_2^2 + T_3^3 + T_4^4 = \rho + 2\xi\theta + \lambda$$

Using co-moving co-ordinates, the field equations based on Lyra geometry (2) together with (3)-(8) for the metric (1) can be obtained as

$$\frac{3\dot{R}^2}{R^2} + \frac{3\dot{R}\dot{S}}{RS} + \frac{3k}{R^2} - \frac{3}{4}\beta^2 = \rho \quad (9)$$

$$\frac{2\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}+\frac{2\dot{R}\dot{S}}{RS}+\frac{\ddot{S}}{S}+\frac{k}{R^2}+\frac{3}{4}\beta^2=\xi\theta \tag{10}$$

$$\frac{3\ddot{R}}{R}+\frac{3\dot{R}^2}{R^2}+\frac{3k}{R^2}+\frac{3}{4}\beta^2=\xi\theta+\lambda \tag{11}$$

The energy conservation equation $T^{j}_{i;j}$ leads to

$$\dot{\rho}+\left(3\frac{\dot{R}}{R}+\frac{\dot{S}}{S}\right)\rho-\lambda\frac{\dot{S}}{S}-\xi(n+3)^2\frac{\dot{R}^2}{R^2}=0 \tag{12}$$

And

$$\left(R^j_i-\frac{1}{2}Rg^j_i\right)_{;j}+\frac{3}{2}\left(\phi_i\phi^j\right)_{;j}-\frac{3}{4}\left(g^j_i\phi_k\phi^k\right)_{;j}=0 \tag{13}$$

which leads to the following equation

$$\frac{3}{2}\beta\dot{\beta}+\frac{3}{2}\beta^2\left(3\frac{\dot{R}}{R}+\frac{\dot{S}}{S}\right)=0 \tag{14}$$

Solution of the Field equations

The field equations (9), (10) and (11) are a system of three independent equations having six unknowns $R, S, \lambda, \rho, \beta$ and ξ. To get the determinate solution, let the deceleration parameter to be a constant[54], i.e.

$$q=-\frac{R\ddot{R}}{\dot{R}^2}=-\frac{\dot{H}+H^2}{H^2}=b \quad \text{(constant)} \tag{15}$$

The above equation may be rewritten as

$$\frac{\ddot{R}}{R}+b\frac{\dot{R}^2}{R^2}=0 \tag{16}$$

On integrating Eqn (16), we get the exact solutions as

$$R(t)=(Ct+D)^{\frac{1}{1+b}}\ ,\quad b\neq-1 \tag{17}$$

$$\text{Or}\ \ R(t)=R_0e^{H_0t}\ ,\quad b=-1$$

where C, D, R_0 and H_0 are constants of integration.

We consider a power law equation because of the existence of anisotropy for the flat and homogeneous universe and $\theta\propto\sigma_{ij}$ (shear tensor). Hence we use the following polynomial relation between the metric co-efficient.

$$S=R^n \tag{18}$$

where n is an arbitrary constant.

Therefore, from (17) and (18), we get

$$S(t)=(Ct+D)^{\frac{n}{1+b}}\ ,\quad b\neq-1 \tag{19}$$

$$\text{Or}\ \ S(t)=R_0e^{nH_0t}\ ,\quad b=-1$$

Case I: $b\neq-1$

The metric in Eqn (1) for FRW model takes the form

$$ds^2=dt^2-(Ct+D)^{\frac{2}{1+b}}\left[\frac{dr^2}{1-kr^2}+r^2\left(d\theta^2+Sin^2\theta d\phi^2\right)\right]-(Ct+D)^{\frac{2n}{1+b}}d\Psi^2 \tag{20}$$

The displacement vector β is obtained as

$$\beta=c_1(Ct+D)^{-\frac{3+n}{1+b}} \tag{21}$$

The expansion scalar θ is obtained as

$$\theta=\frac{C(n+3)}{(1+b)(Ct+D)} \tag{22}$$

From Eqn (9), we get

$$\rho=\frac{3C^2(n+1)}{(1+b)^2(Ct+D)^2}+\frac{1}{4}(Ct+D)^{-\frac{2(3+n)}{1+b}}\left[12k(Ct+D)^{\frac{2(2+n)}{1+b}}-3{c_1}^2\right] \tag{23}$$

From Eqn (10), we get

$$\xi\theta=\frac{4C^2-8bC^2+8nC^2-4nC^2(1+b-n)+4k(1+b)^2(Ct+D)^{\frac{2b}{1+b}}+3{c_1}^2(1+b)^2(Ct+D)^{2-\frac{2(3+n)}{1+b}}}{4(1+b)^2(Ct+D)^2}$$

$$\xi=\frac{4C^2-8bC^2+8nC^2-4nC^2(1+b-n)+4k(1+b)^2(Ct+D)^{\frac{2b}{1+b}}+3{c_1}^2(1+b)^2(Ct+D)^{2-\frac{2(3+n)}{1+b}}}{4C(n+3)(1+b)(Ct+D)} \tag{24}$$

From Eqn (11), we get

$$\lambda=\frac{4C^2-10bC^2+4nC^2-2nC^2(1+b-n)+8k(1+b)^2(Ct+D)^{\frac{2b}{1+b}}+3{c_1}^2(1+b)^2(Ct+D)^{2-\frac{2(3+n)}{1+b}}}{2(1+b)^2(Ct+D)^2} \tag{25}$$

From (23) and (25) together with (6), we get

$$\rho_p=\frac{4C^2-20bC^2+20nC^2-4nC^2(1+b-n)+28k(1+b)^2(Ct+D)^{\frac{2b}{1+b}}+3{c_1}^2(1+b)^2(Ct+D)^{2-\frac{2(3+n)}{1+b}}}{4(1+b)^2(Ct+D)^2} \tag{26}$$

I. $k=0$, Flat model

In this Particular case, the model becomes

$$ds^2 = dt^2 - (Ct + D)^{\frac{2}{1+b}}[dr^2 + r^2(d\theta^2 + Sin^2\theta d\phi^2)] - (Ct + D)^{\frac{2n}{1+b}} d\Psi^2 \quad (27)$$

The energy density for this model is given by

$$\rho = \frac{3C^2(n+1)}{(1+b)^2(Ct+D)^2} - \frac{3}{4}(Ct + D)^{-\frac{2(3+n)}{1+b}} c_1^{\,2} \quad (28)$$

The bulk Viscosity is

$$\xi = \frac{4C^2 - 8bC^2 + 8nC^2 - 4nC^2(1+b-n) + 3c_1^{\,2}(1+b)^2 (Ct+D)^{2-\frac{2(3+n)}{1+b}}}{4C(n+3)(1+b)(Ct+D)} \quad (29)$$

The tension density is given by

$$\lambda = \frac{4C^2 - 10bC^2 + 4nC^2 - 2nC^2(1+b-n) + 3c_1^{\,2}(1+b)^2 (Ct+D)^{2-\frac{2(3+n)}{1+b}}}{2(1+b)^2(Ct+D)^2} \quad (30)$$

The particle density is

$$\rho_p = \frac{4C^2 - 20bC^2 + 20nC^2 - 4nC^2(1+b-n) + 3c_1^{\,2}(1+b)^2 (Ct+D)^{2-\frac{2(3+n)}{1+b}}}{4(1+b)^2(Ct+D)^2} \quad (31)$$

II. **$k = 1$, Closed model**

The metric for FRW model takes the form

$$ds^2 = dt^2 - (Ct + D)^{\frac{2}{1+b}}\left[\frac{dr^2}{1-r^2} + r^2(d\theta^2 + Sin^2\theta d\phi^2)\right] - (Ct + D)^{\frac{2n}{1+b}} d\Psi^2 \quad (32)$$

The energy density for this model is given by

$$\rho = \frac{3C^2(n+1)}{(1+b)^2(Ct+D)^2} + \frac{1}{4}(Ct + D)^{-\frac{2(3+n)}{1+b}} \left[12(Ct + D)^{\frac{2(2+n)}{1+b}} - 3c_1^{\,2}\right] \quad (33)$$

The bulk Viscosity is

$$\xi = \frac{4C^2 - 8bC^2 + 8nC^2 - 4nC^2(1+b-n) + 4(1+b)^2(Ct+D)^{\frac{2b}{1+b}} + 3c_1^{\,2}(1+b)^2(Ct+D)^{2-\frac{2(3+n)}{1+b}}}{4C(n+3)(1+b)(Ct+D)} \quad (34)$$

The tension density is given by

$$\lambda = \frac{4C^2 - 10bC^2 + 4nC^2 - 2nC^2(1+b-n) + 8(1+b)^2(Ct+D)^{\frac{2b}{1+b}} + 3c_1^{\,2}(1+b)^2(Ct+D)^{2-\frac{2(3+n)}{1+b}}}{2(1+b)^2(Ct+D)^2} \quad (35)$$

The particle density is

$$\rho_p = \frac{4C^2 - 20bC^2 + 20nC^2 - 4nC^2(1+b-n) + 28(1+b)^2(Ct+D)^{\frac{2b}{1+b}} + 3c_1^{\,2}(1+b)^2(Ct+D)^{2-\frac{2(3+n)}{1+b}}}{4(1+b)^2(Ct+D)^2} \quad (36)$$

I. **$k = -1$, Open model**

The metric for FRW model takes the form

$$ds^2 = dt^2 - (Ct + D)^{\frac{2}{1+b}}\left[\frac{dr^2}{1+r^2} + r^2(d\theta^2 + Sin^2\theta d\phi^2)\right] - (Ct + D)^{\frac{2n}{1+b}} d\Psi^2 \quad (37)$$

The energy density for this model is given by

$$\rho = \frac{3C^2(n+1)}{(1+b)^2(Ct+D)^2} + \frac{1}{4}(Ct + D)^{-\frac{2(3+n)}{1+b}} \left[-12(Ct + D)^{\frac{2(2+n)}{1+b}} - 3c_1^{\,2}\right] \quad (38)$$

The bulk Viscosity is

$$\xi = \frac{4C^2 - 8bC^2 + 8nC^2 - 4nC^2(1+b-n) - 4(1+b)^2(Ct+D)^{\frac{2b}{1+b}} + 3c_1^{\,2}(1+b)^2(Ct+D)^{2-\frac{2(3+n)}{1+b}}}{4C(n+3)(1+b)(Ct+D)} \quad (39)$$

The tension density is given by

$$\lambda = \frac{4C^2 - 10bC^2 + 4nC^2 - 2nC^2(1+b-n) - 8(1+b)^2(Ct+D)^{\frac{2b}{1+b}} + 3c_1^{\,2}(1+b)^2(Ct+D)^{2-\frac{2(3+n)}{1+b}}}{2(1+b)^2(Ct+D)^2} \quad (40)$$

The particle density is

$$\rho_p = \frac{4C^2 - 20bC^2 + 20nC^2 - 4nC^2(1+b-n) - 28(1+b)^2(Ct+D)^{\frac{2b}{1+b}} + 3c_1^{\,2}(1+b)^2(Ct+D)^{2-\frac{2(3+n)}{1+b}}}{4(1+b)^2(Ct+D)^2} \quad (41)$$

3.2. Case II: $b = -1$

The metric in Eqn (1) for FRW model takes the form

$$ds^2 = dt^2 - R_0^2 e^{2H_0 t}\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + Sin^2\theta d\phi^2)\right] - R_0^{2n} e^{2nH_0 t} d\Psi^2 \quad (42)$$

The displacement vector β is obtained as

$$\beta = c_0 e^{-(n+3)H_0 t} \quad (43)$$

The expansion scalar θ is

$$\theta = (n+3)H_0 \quad (44)$$

From Eqn (9), we get

$$\rho = 3(n+1)H_0^2 + \frac{3k}{R_0^2 e^{2H_0 t}} - \frac{3}{4}c_0^2 e^{-2(n+3)H_0 t} \quad (45)$$

From Eqn (10), we get

$$\xi\theta = (n^2+2n+3)H_0^2 + \frac{k}{R_0^2 e^{2H_0 t}} - \frac{3}{4}c_0^2 e^{-2(n+3)H_0 t}$$

$$\xi = \frac{(n^2+2n+3)H_0^2 + \frac{k}{R_0^2 e^{2H_0 t}} - \frac{3}{4}c_0^2 e^{-2(n+3)H_0 t}}{(n+3)H_0} \quad (46)$$

From Eqn (11), we get

$$\lambda = (n^2+2n+9)H_0^2 + \frac{4k}{R_0^2 e^{2H_0 t}} \quad (47)$$

The particle density is

$$\rho_p = 3(5n-6)H_0^2 + \frac{7k}{R_0^2 e^{2H_0 t}} - \frac{3}{4}c_0^2 e^{-2(n+3)H_0 t} \quad (48)$$

I. $k = 0$, Flat model

The metric for FRW model takes the form

$$ds^2 = dt^2 - R_0^2 e^{2H_0 t}[dr^2 + r^2(d\theta^2 + Sin^2\theta d\phi^2)] - R_0^{2n} e^{2nH_0 t} d\Psi^2 \quad (49)$$

The expansion scalar θ is

$$\theta = (n+3)H_0 \quad (50)$$

The Energy density is

$$\rho = 3(n+1)H_0^2 - \frac{3}{4}c_0^2 e^{-2(n+3)H_0 t} \quad (51)$$

The bulk Viscosity is

$$\xi = \frac{(n^2+2n+3)H_0^2 - \frac{3}{4}c_0^2 e^{-2(n+3)H_0 t}}{(n+3)H_0} \quad (52)$$

The String tension Density is

$$\lambda = (n^2+2n+9)H_0^2 \quad (53)$$

The particle density is

$$\rho_p = 3(5n-6)H_0^2 - \frac{3}{4}c_0^2 e^{-2(n+3)H_0 t} \quad (54)$$

I. $k = 1$, Closed model

The metric for FRW model takes the form

$$ds^2 = dt^2 - R_0^2 e^{2H_0 t}\left[\frac{dr^2}{1-r^2} + r^2(d\theta^2 + Sin^2\theta d\phi^2)\right] - R_0^{2n} e^{2nH_0 t} d\Psi^2 \quad (55)$$

The Energy density is

$$\rho = 3(n+1)H_0^2 + \frac{3}{R_0^2 e^{2H_0 t}} - \frac{3}{4}c_0^2 e^{-2(n+3)H_0 t} \quad (56)$$

The bulk Viscosity is

$$\xi = \frac{(n^2+2n+3)H_0^2 + \frac{1}{R_0^2 e^{2H_0 t}} - \frac{3}{4}c_0^2 e^{-2(n+3)H_0 t}}{(n+3)H_0} \quad (57)$$

The string tension density is

$$\lambda = (n^2+2n+9)H_0^2 + \frac{4}{R_0^2 e^{2H_0 t}} \quad (58)$$

The particle density is

$$\rho_p = 3(5n-6)H_0^2 + \frac{7}{R_0^2 e^{2H_0 t}} - \frac{3}{4}c_0^2 e^{-2(n+3)H_0 t} \quad (59)$$

II. $k = -1$, Open model

The metric for FRW model takes the form

$$ds^2 = dt^2 - R_0^2 e^{2H_0 t}\left[\frac{dr^2}{1+r^2} + r^2(d\theta^2 + Sin^2\theta d\phi^2)\right] - R_0^{2n} e^{2nH_0 t} d\Psi^2 \quad (60)$$

The energy density is

$$\rho = 3(n+1)H_0^2 - \frac{3}{R_0^2 e^{2H_0 t}} - \frac{3}{4}c_0^2 e^{-2(n+3)H_0 t} \quad (61)$$

The bulk Viscosity is

$$\xi = \frac{(n^2+2n+3)H_0^2 - \frac{1}{R_0^2 e^{2H_0 t}} - \frac{3}{4}c_0^2 e^{-2(n+3)H_0 t}}{(n+3)H_0} \quad (62)$$

The tension density is

$$\lambda = (n^2+2n+9)H_0^2 - \frac{4}{R_0^2 e^{2H_0 t}} \quad (63)$$

The particle density is

$$\rho_p = 3(5n-6)H_0^2 - \frac{7}{R_0^2 e^{2H_0 t}} - \frac{3}{4}c_0^2 e^{-2(n+3)H_0 t} \quad (64)$$

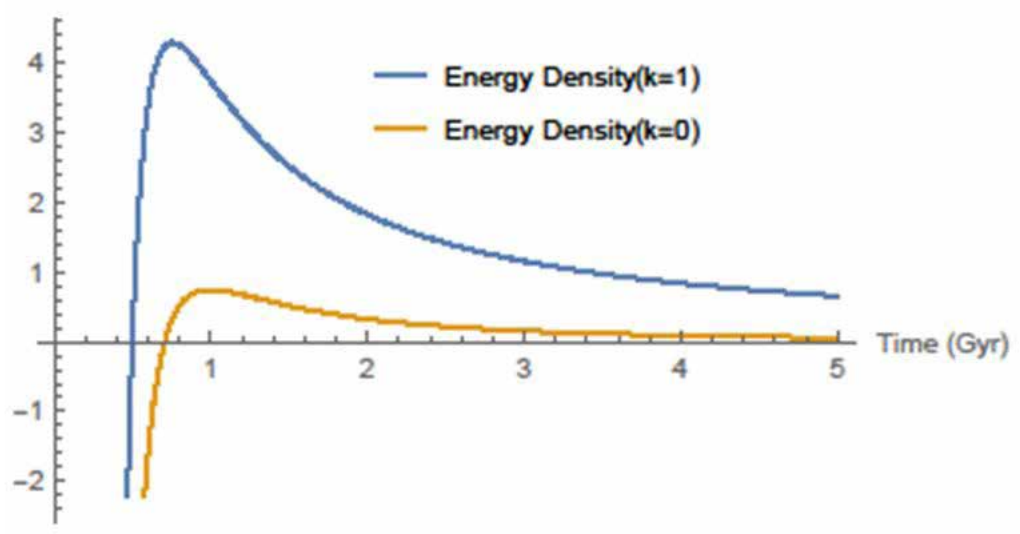

Figure 1: Variation of ρ with t in Gyr for $b \neq -1$

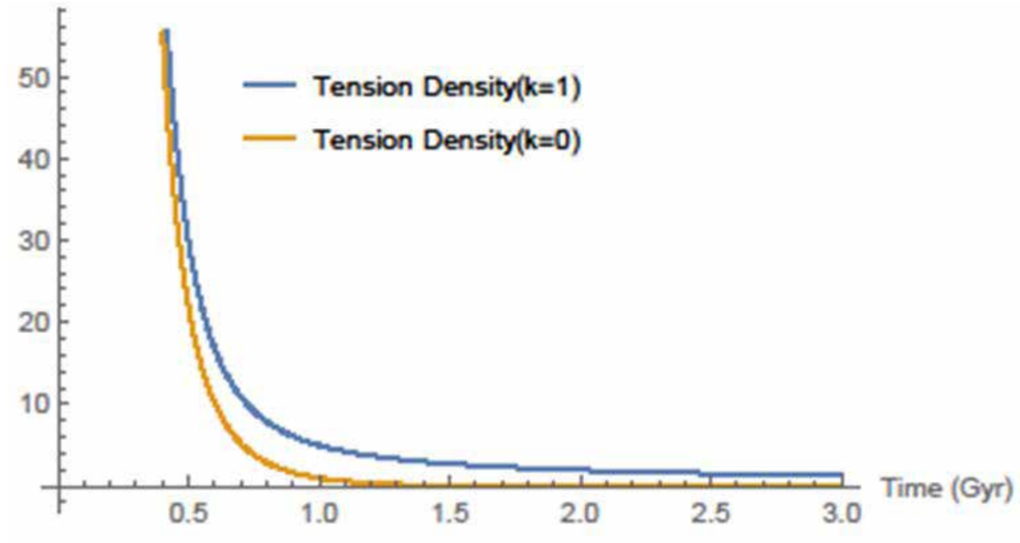

Figure 2: Variation of λ with t in Gyr for $b \neq -1$

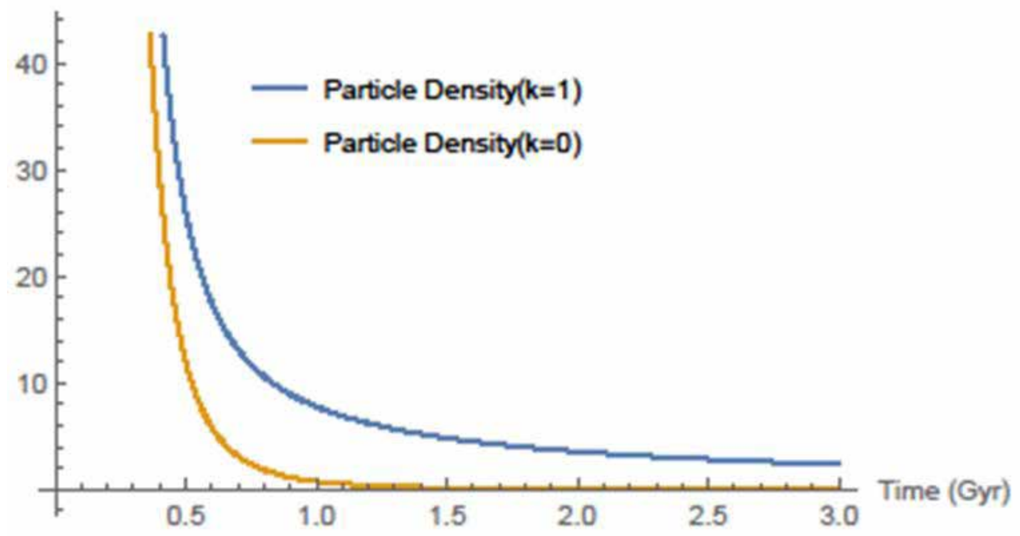

Figure 3: Variation of ρ_p, with t in Gyr for $b \neq -1$

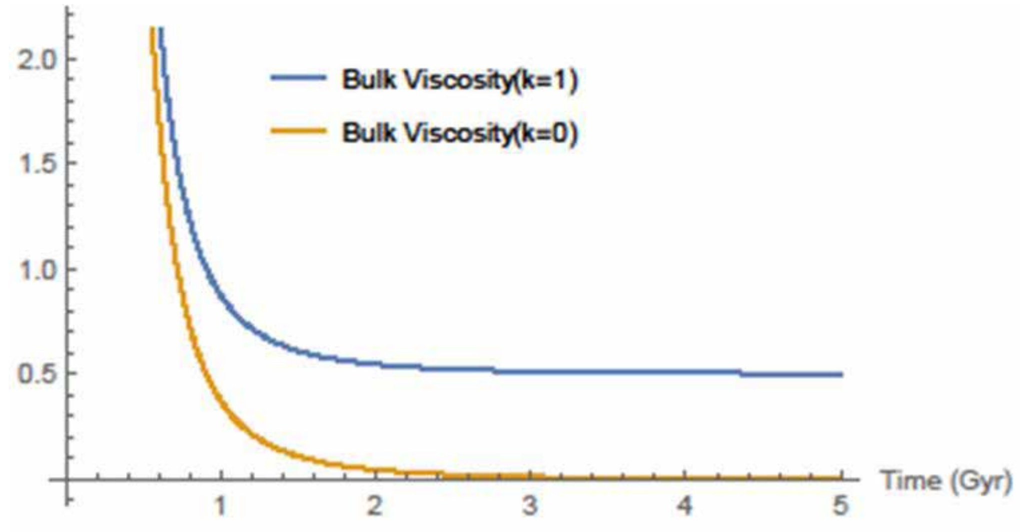

Figure 4: Variation of ξ with t in Gyr for $b \neq -1$

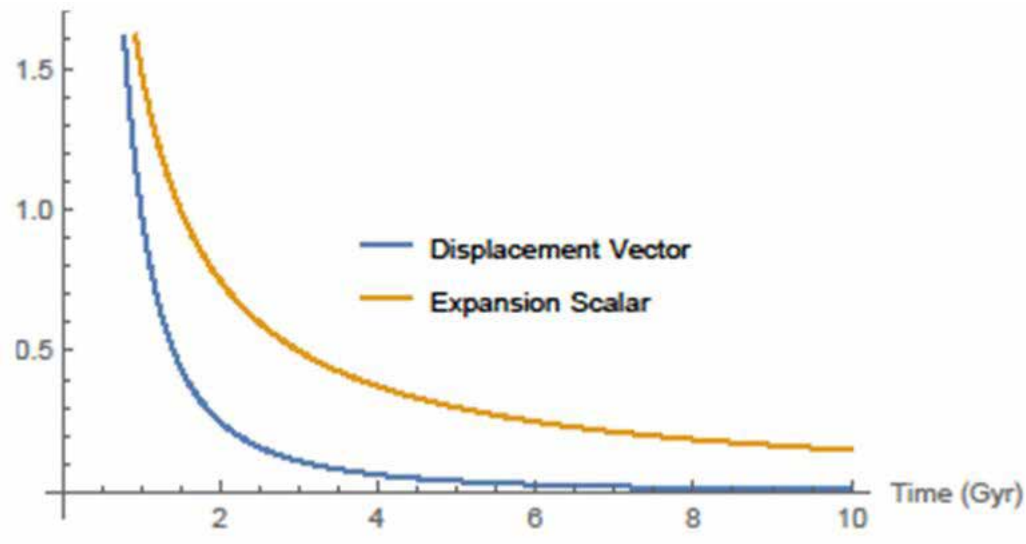

Figure 5: Variation of β,θ with t in Gyr for $b \neq -1$

Physical interpretations of the models

The physical parameters have been plotted for flat and closed Universe. However, the model (k = –1) is not suitable to explain the current Universe. Figure 1 depicts the behaviour of energy density versus time. We observed that ρ changes sign from negative to positive value after some finite time and reaches large value and decreases with the time approaching a small positive value at late times for closed model (k = –1). For a flat model (k = 0), the energy density is negative at the initial epoch and maintains a small positive value at an early stage, and is almost coincident with zero at late times. Both particle density and string tension density are always positive in the closed model (k = 1) whereas, they decrease more sharply with the cosmic time and approaches zero in the flat model (k = 0) [Figures 2 and 3]. It is observed that the string tension density disappears more rapidly than particle density leaving particles only indicating the matter dominated universe at late times as anticipated. The variation of bulk viscosity ξ with time is shown in figure 4. In the flat model (k = 0), the bulk viscosity ξ decreases with time leading to an inflationary model and vanishes for infinitely large time t. In the closed model (k = 1), ξ decreases, remain positive throughout the evolution. The function of the bulk viscosity is to retard the expansion of the universe and since bulk viscosity decreases with time, retardedness also decreases which supports the expansion at a faster rate in the late stages of the evolution of the universe. The displacement vector β and the expansion scalar θ have been found out for flat and closed models and are plotted in figure 5. We noticed that β and θ decrease with the increase in the age of the universe. At the initial epoch of time, the gauge function β^2 is found to be infinite and ultimately $\beta^2 \to 0$ when $t \to \infty$.

Conclusions

In the present study, we investigated bulk viscous fluid attached to the string cloud by considering a time-dependent deceleration parameter for a five-dimensional FRW universe in the context of Lyra Geometry. Here, we restricted our study to a constant deceleration parameter as predicted from observation [55]. The solutions of the model have been obtained for flat, closed, and open bulk viscous string FRW universes in five dimensions. The physical parameters have been plotted for $b \neq -1$. However, in the case of $b = -1$, all the parameters vanish rapidly within a short period of time. This fact indicates that the solution represents an early era of the evolution of the universe. The incorporation of bulk viscosity in our investigation is to replace the condition of material distribution other than perfect fluid. The bulk viscosity plays a significant role in the present scenario of the evolution of the universe. So, our model will contribute to a better understanding of spatially homogeneous and isotropic accelerating universe [56] in five dimensions.

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