Scientific Eminence Group

Research Article

Open Access

Journal of Nuclear Science and Technology

jnst@scientificeminencegroup.com

Influence of Modulated Dzyaloshinskii-Moriya Interaction on the Bipartite and Tripartite Quantum Coherence of 1/2-XXZ Heisenberg Spin Induce Electric Polarization in the Chain

G. C. Fouokeng*, C. Tahabo Ngueldjou and M. Tchoffo

Condensed Matter, Electronics and Signal Processing Research Unit, Faculty of Sciences, University of Dschang, P.O. Box 67 Dschang, Cameroon

***Corresponding Author**

G. C. Fouokeng, Condensed Matter, Electronics and Signal Processing Research Unit, Faculty of Sciences, University of Dschang, P.O. Box 67 Dschang, Cameroon, E-mail: fouokenggc2012@yahoo.fr

Citation

G. C. Fouokeng, C. Tahabo Ngueldjou, M. Tchoffo (2025) Influence of modulated Dzyaloshinskii-Moriya interaction on the bipartite and tripartite quantum coherence of 1/2-XXZ Heisenberg spin induce electric polarization in the chain. J Nucl Sci Tech. 1: 103

Publication Dates

Received date: May 02, 2025
Accepted date: May 20, 2025
Published date: May 27, 2025

Abstract

A bipartite and tripartite quantum coherence of a polarized 1/2–XXZ Heisenberg spins chain model with modulated Dzyaloshinskii-Moriya interaction and applied magnetic field is investigated using quantum renormalization group approaches. In a thermodynamic limits, we derived the coherence factor and the monogamy of entanglement of a bipartite and tripartite states of the modulated system. Our analysis show how much the quantum coherence and the monogamy of entanglement decrease in the weak regime of the external magnetic field with the increases of the anisotropy factor and the decreases of the Dzyaloshinsky-Moriya factor D_z. We observe an interplay between the anisotropy factor and the applied magnetic field and the modulated DM factor. The combined effect of the modulate DM interaction with the applied magnetic field bring correction to the non-analyticity behavior of quantum coherence and of monogamy of entanglement exhibited around the criticalpoint $\triangle = 1$ of the anisotropy factor. In addition, we perceive that the non-analytical behavior around the critical value of the anisotropy factor translate quantum phase transition from spin liquid to N´eel phase and vice-versa depending on whether spin is localised on an even or odd site in the chain. The electric field polarisation induced in the chain tens to stabilize the spins in a specific orientations and reenforce correlations which becomes beneficial for ferroelectricity and opened route to switch magnetoelectricity with electric polarazation and vice-versa.

Keywords: Electric Field Polarisation; Bipartite State; Tripartite State; Quantum Coherence; Modulated Dzyaloshinskii-Moriya interaction.

Introduction

Many-body systems have been attracting growing interest in semiconductor physics for several decades, particularly when interested in, quantum phase transition (QPT) [1–7] and technology of quantum system with highest entangled and strong correlated sates [8–15]. Several theoretical works have been done in this aim using different scenarios, among which the study of two potential quantifiers of quantum coherence in a physical state characterized by the ℓ1-norm of the system density matrix and the quantum relative entropy with respect to a diagonal matrix [16]. The dynamic of information in quantum systems, have been linked to quantum coherence trough quantum correlations measurements parameters by making used of establishment of an hier archical and explicit relationship between quantum coherence, quantum discord and entanglement between quantum states of a multi-particle system [7,17,18]. However, due to the elastic dimension of the Hilbert space of many-body systems, the analytical and exact study of quantum correlations by the ordinary mathematical approaches become complex. That is, the quantum renormalization group (QRG) approaches has been proposed as well-done approach [19–21]. For example, the Kadanoff QRG of spin squeezing approaches (which consists of dividing spin chain into an intra-block (HB) and inter-block (HBB) components) have been used as an indicator of QPT[22]. The non-analytic behavior at the critical points (translating a transition from the spin liquid phase to the N´eel phase) depending on the anisotropy of the spin chain have been obtained as a signature of quantum phase transition [18,21,22] leading also to quantum coherence lost, which becomes prominent with the dimension of the Hilbert space defined by different iterations of the QRG. on the overview of the considered method, some authors have investigate quantum coherence dynamics in an Ising transverse field model (ITF) and in an Heisenberg XXZ model, with and without Dzyaloshinskii Moriya (DM) interaction on a periodic spin chain of N lattice sites. They showed that the spin compression parameter obtained after each step of the QRG widened when the scale of the system becomes larger; Similarly, as a function of the QRG iteration number, the different ground state (GS) spin compression parameters exhibited an abrupt change in concavity with a discontinuity in the behavior of the first derivatives of these parameters, characterizing QPT signature [18,21,22].

In the light of the work done by joya in 2021 and Balazadeh in 2018[18,21], our objective in this paper is to evaluate the contribution of the electric polarization and of the Modulated Dzyaloshinskii-Moriya Interaction (DMI)on the non-analytical dynamics of and entangled spin states in Heisenberg spin-1/2 xxz chain using QRG method. Our consideration aims to maintain quantum correlations and entanglement as long as possible.

In order to improve the transport properties in spin chains, the problem of coupling between states of the system is growing, particularly when interested to quantum coherence or quantum phase transition. Among other coupling possibilities, the spinorbit coupling [23,24] particularly in magnetic thin layers, between spins, between orbital moments and degrees of freedom related to the structure of materials as well as the interactions between systems and external excitation field [25,26]show better parameters. An interest is also often placed on the types of interface and surface states as well as on the exotic spin states [28]. The study of DM interaction in thin film systems has shown an even greater interest in spin electronics, which has motivated the exploitation of the strength of its intensity for the development of even more functional materials [29,30]. However, this study only concerned systems whose interactions are limited to the first neighbors, but with vector spin variables on each site of the network. This is how the modulation effects coming from the competition between the usual Heisenberg exchange, scalar in nature, and an additional exchange associated with the vector products of DM interactions have been taken into account [30,32,33].

Beyond taking into account the modulated DM interaction in spin chains, the interest of which has already been demonstrated, we wish to go further by considering in our work the effect of a polarization induced by the electric field on the dynamics of the bipartite and tripartite states of a 1/2 − XXZ Heisenberg renormalised spin chain.

To achieve the goal of this work, the paper will be organized as follows: the second part will present the quantum model as well as the method used for the calculation of the descriptive parameters of the system. The third part will present the analytical an numerical results with the different interpretations; the work will end with a conclusion and perspectives.

XXZ Model and Application of Quantum Renormalization Group Approach

The model used for this purpose is that of a long and periodic chain of 1/2-XXZ Heisenberg model in the presence of both modulated Dzyaloshinskii-Moriya interaction and constant external magnetic field, under the influence of an external electrical polarization (see Eq. (1), with h = 1 for simplicity):

$$H = H_{XXZ} + H_{DM} + H_{Pol} \quad (1)$$

where

$$H_{XXZ} = \sum_{n=1}^{N} \left[J \left(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y \right) + J_z S_n^z S_{n+1}^z - h(n) S_n^z \right] \quad (2)$$

is the XXZ part hamiltonian, $J_{i=x,y,z} > 0$ the antiferromagnetic coupling term between the spins along the X, Y and Z directions. The Hamiltonian of the Dzyaloshinskii-Moriya interaction modulated by the spinchain position dependence is [32]:

$$H_{DM} = \sum_{n=1}^{N} \vec{d}(n) \bullet \left(\vec{S}_n \times \vec{S}_{n+1} \right) \quad (3)$$

The contribution of the bias field is given by:

$$H_{Pol} = -\vec{E} \bullet \vec{P} \quad (4)$$

where $\vec{E} = (0,E, 0)$ is a constant electric field oriented along $\vec{e}_y$ and

$$\vec{P} = \frac{1}{N} \sum_{n=1}^{N} \left[\vec{e}_x \times \left(\vec{S}_n \times \vec{S}_{n+1} \right) \right] \quad (5)$$

the polarization vector, with S^i_n the spin operator. The model described in Eq. (1) is spin-position dependent according to the external magnetic field h(n) and the Dzyaloshinskii Moriya interaction coefficient d(n) which are the function of the spin position in the chain defined through < n >:

$$h(n) = h_0 + (-1)^n h_1 \quad (6)$$

$$\vec{d}(n) = (d_0 + (-1)^n d_1)\, \vec{e}_z \quad (7)$$

With the help of the Bloch relation and the RG approaches, the characteristic Hamiltonian of the spin chain in Eq. (1) can be transformed into the following form [7,18]:

$$H = H^B + H^{BB} \tag{8}$$

respectively with

$$H^B = \frac{J}{4}\sum_{l=1}^{N/3}\left[\begin{array}{l}\sigma^x_{1,l}\sigma^x_{2,l} + \sigma^x_{2,l}\sigma^x_{3,l} + \sigma^y_{1,l}\sigma^y_{2,l} + \sigma^y_{2,l}\sigma^y_{3,l} + \Delta\left(\sigma^z_{1,l}\sigma^z_{2,l} + \sigma^z_{2,l}\sigma^z_{3,l}\right)\\ -\tilde{h}(n)\left(\sigma^z_{1,l} + \sigma^z_{2,l} + \sigma^z_{3,l}\right) + (D(n)+\varepsilon)\left(\sigma^x_{1,l}\sigma^y_{2,l} - \sigma^y_{1,l}\sigma^x_{2,l} + \sigma^x_{2,l}\sigma^y_{3,l} - \sigma^y_{2,l}\sigma^x_{3,l}\right)\end{array}\right] \tag{9}$$

the Hamiltonian of the block of the intra-block and

$$H^{BB} = \frac{J}{4}\sum_{l=1}^{N/3}\left[\begin{array}{l}\sigma^x_{3,l}\sigma^x_{1,l+1} + \sigma^y_{3,l}\sigma^y_{1,l+1} + \Delta\left(\sigma^z_{3,l}\sigma^z_{1,l+1}\right) - \tilde{h}(n)\left(\sigma^z_{3,l}\sigma^z_{1,l+1}\right) +\\ +(D(n)+\varepsilon)\left(\sigma^x_{3,l}\sigma^y_{1,l+1} - \sigma^y_{3,l}\sigma^x_{1,l+1}\right)\end{array}\right] \tag{10}$$

the Hamiltonian of the inter-block i.e that reflecting the interaction between the block l and the block $l + 1$, With l the rank of the block; where σ^2_n in is the Pauli matrice, $\triangle = J_z/j$the anisotropy factor and ξ = E JN the electric polarization factor. ˜h(n) = 2h(n) J and D(n) = d(n) J are the new quantities of both modulated external magnetic field and DM Interaction coefficient.

In order to evaluate the fundamental parameters of the system in application of the GR-principle, the operator of projection ζ is constructed by the lower eigenvectors, that is to say, those which derive the energy from its fundamental state:

$$\zeta = \sum_{l=1}^{N/3}\zeta_0^l \tag{11}$$

where

$$\zeta_0^l = |\Uparrow\rangle_l \langle\Phi_0| + |\Downarrow\rangle_l \langle\Phi'_0| \tag{12}$$

the kets $|\Uparrow i l$ and $|\Downarrow i l$ are the new states in l –th bloc of spin defined in the renormalized Hilbert space obeying the same properties as those of the spin-1/2 ; $\emptyset_0|$ ($\emptyset'_0|$) are eigen vectors of the fundamental state. The energy of the fundamental state. The energy of the fundamental state

$$E_0 = -\frac{J}{4}\left(\Delta + \tilde{h}(n) + \sqrt{\Delta^2 + 8\left(1 + (D(n)+\varepsilon)^2\right)}\right)$$

from Eq. (10) of the l-th block is doubly degenerate so that

$$\begin{array}{l}|\Phi_0\rangle = \alpha\,|\downarrow\downarrow\uparrow\rangle + \beta\,|\downarrow\uparrow\downarrow\rangle + \gamma\,|\uparrow\downarrow\downarrow\rangle\,,\\ |\Phi'_0\rangle = \alpha\,|\downarrow\uparrow\uparrow\rangle + \beta\,|\uparrow\downarrow\uparrow\rangle + \gamma\,|\uparrow\uparrow\downarrow\rangle\end{array} \tag{13}$$

where $|\uparrow i$ and $|\downarrow i$ are eigenstates of the Pauli matrix σ^z. The parameters α , β , γ that verify the property $\alpha 2+\beta 2 + \gamma 2 = 1$, can be defined by the following relations.

$$\begin{array}{l}\alpha = 2\left((D(n)+\varepsilon)^2 + 1\right);\\ \beta = -\left(1 - i\,(D(n)+\varepsilon)\right)(\Delta + q)\,;\\ \gamma = -2\left(2i\,(D(n)+\varepsilon) + \left((D(n)+\varepsilon)^2 - 1\right)\right)\end{array} \tag{14}$$

with

$$q = \sqrt{\Delta^2 + 8\left(1 + (D(n)+\varepsilon)^2\right)}$$

Let $\zeta\dagger$ be the transpose complex conjugate of the projection operator ζ , the effective Hamiltonian of the system built around the total Hamiltonian of the system Eq. (8) and which acts on the renormalized subspace through the relation

$$H_{eff} = \zeta H \zeta^{\dagger} \tag{15}$$

Leads to relationship:

$$H_{eff}{}^{(p+1)}(n) = \frac{H_{eff}{}^{(p)}(n)\left(\sqrt{\left(\Delta^{(p)}\right)^2 + 8\left(1 + \left(D^{(p)}(n) + \varepsilon^{(p)}\right)^2\right)}\right)^2}{4\left(1 + \left(D^{(p)}(n) + \varepsilon^{(p)}\right)^2\right)} \tag{16}$$

where p is the order of the renormalization, $D^{(p+1)}(n) = D^{(p)}(n) = D(n)$ and $\varepsilon^{(p+1)} = \varepsilon^{(p)} = \varepsilon$. From the first renormalisation i.e p = 1 the hamiltonian Eq. (16) gives:

$$H_{eff} = \sum_{l=1}^{N/3} \frac{J'}{4}\left[\tilde{\sigma}_l^x \tilde{\sigma}_{l+1}^x + \tilde{\sigma}_l^y \tilde{\sigma}_{l+1}^y + \Delta' \tilde{\sigma}_l^z \tilde{\sigma}_{l+1}^z - \tilde{h}'(n)\tilde{\sigma}_l^z + \left(D'(n) + \varepsilon'\right)\left(\tilde{\sigma}_l^x \tilde{\sigma}_{l+1}^y - \tilde{\sigma}_l^y \tilde{\sigma}_{l+1}^x\right)\right] \tag{17}$$

With the following new parameters $J' = \frac{4J\left(1+(D(n)+\varepsilon)^2\right)}{q^2}$; $\Delta' = \frac{\Delta(\Delta+q)^2}{16\left(1+(D(n)+\varepsilon)^2\right)}$; $D'(n) = D(n)$; $\varepsilon' = \varepsilon$ and $\tilde{h}'(n) = \frac{q^2\tilde{h}(n)}{4\left(1+(D(n)+\varepsilon)^2\right)}$. These relations describe the iteration relations between the parameters of the original system and those of the first step of the RG.

Here, the dynamics of Bipartite and tripartite quantum information resources under the effect of the anisotropy parameters $\triangle'$, the modulated Dzyaloshinskii interaction $D'(n)$, the electric polarization coefficient ε' and of the modulated magnetic field $\tilde{h}'(n)$ is measured trough quantum coherence and the Monogamy of entanglement, as well as Polygamy of entanglement, by using the ℓ_1-norm of density matrix approximation.

Considering one of the degenerate ground states of the system obtained in Eq. (13)), the initial density matrix $\rho_0^{123} = |\emptyset_0)(\emptyset_{0|}$ of an uncorrelated states of three spins located in a block defined :

$$\rho_0^{123} = \frac{1}{|\alpha|^2 + |\beta|^2 + |\gamma|^2}\begin{bmatrix} |\alpha|^2 |\downarrow\downarrow\uparrow\rangle\langle\downarrow\downarrow\uparrow| + \alpha\beta^* |\downarrow\downarrow\uparrow\rangle\langle\downarrow\uparrow\downarrow| + \alpha\gamma^* |\downarrow\downarrow\uparrow\rangle\langle\uparrow\downarrow\downarrow| \\ +\beta\alpha^* |\downarrow\uparrow\downarrow\rangle\langle\downarrow\downarrow\uparrow| + |\beta|^2 |\downarrow\uparrow\downarrow\rangle\langle\downarrow\uparrow\downarrow| + \beta\gamma^* |\downarrow\uparrow\downarrow\rangle\langle\uparrow\downarrow\downarrow| \\ +\gamma\alpha^* |\uparrow\downarrow\downarrow\rangle\langle\downarrow\downarrow\uparrow| + \gamma\beta^* |\uparrow\downarrow\downarrow\rangle\langle\downarrow\uparrow\downarrow| + |\gamma|^2 |\uparrow\downarrow\downarrow\rangle\langle\uparrow\downarrow\downarrow| \end{bmatrix} \tag{18}$$

In the ℓ_1-norm approximation [16] the closed form of the quantum coherence measurment factor is given by

$$C_{l1}(\rho) = \sum_{i \neq i} |\langle i| \rho_{ij} |j\rangle| \tag{19}$$

This factor is the sum of the absolute values of the off-diagonal elements of the density matrix. |i) and |j)are the basis vectors of Hilbert space. For an initial states, Eq. (19)) can be rewrite:

$$C_{\ell_1}(\rho_0) = \sum_{i,j;i\neq j} \lfloor \rho_{i,j} \lfloor \tag{20}$$

with $\rho_{i,j}$ the density matrix of the initial states define by:

$$\rho_0^{ij} = Tr_k\left(\rho_0^{ijk}\right) \tag{21}$$

where *i, j, k* represents the positions of the spins in the block.

Depending on different and possible permutations of the spin positions in the chain, only the formation of three bipartite states $C_{l1}(\rho_0^{ij})$ and one tripartite states are possible(with $i \neq j \neq k$ and can occupy only the following distinguish positions 1, 2, 3).

Dynamics of Bipartite States

The density matrix that characterizes the collective behavior of the two spins between the three considered spin at different positions *i, j, k* in the block is derived by partial tracing on the third as $\rho^{ij}0 = T_{rk}(\rho^{ijk}0)$. Thus, for the 1st and 2nd spin of one bloc, the density matrix(eqn.(22))

$$\rho_0^{12} = \frac{1}{|\alpha|^2 + |\beta|^2 + |\gamma|^2} \left[\begin{array}{l} |\alpha|^2 |\downarrow\downarrow\rangle \langle\downarrow\downarrow| + |\beta|^2 |\downarrow\uparrow\rangle \langle\downarrow\uparrow| + |\gamma|^2 |\uparrow\downarrow\rangle \langle\uparrow\downarrow| \\ +\gamma\beta^* |\uparrow\downarrow\rangle \langle\downarrow\uparrow| + \beta\gamma^* |\downarrow\uparrow\rangle \langle\uparrow\downarrow| \end{array} \right] \tag{22}$$

is the bipartite density matrix for spins at positions 1, 2. Considering Eq. (22)) into Eq. (21)), the quantization parameters of bipartite quantum coherence is obtained:

$$C_{l1}\left(\rho_0^{12}\right) = C_{l1}\left(\rho_0^{23}\right) = \frac{2|\beta||\gamma|}{|\alpha|^2 + |\beta|^2 + |\gamma|^2} = \frac{2\sqrt{(D(n)+\varepsilon)^2 + 1}}{q\left(|\alpha|^2 + |\beta|^2 + |\gamma|^2\right)} \tag{23}$$

$$C_{l1}\left(\rho_0^{13}\right) = \frac{2|\alpha||\gamma|}{|\alpha|^2 + |\beta|^2 + |\gamma|^2} = \frac{2\left((D(n)+\varepsilon)^2 + 1\right)}{q(q+\Delta)\left(|\alpha|^2 + |\beta|^2 + |\gamma|^2\right)} \tag{24}$$

Similarly, the tripartite quantum coherence quantization

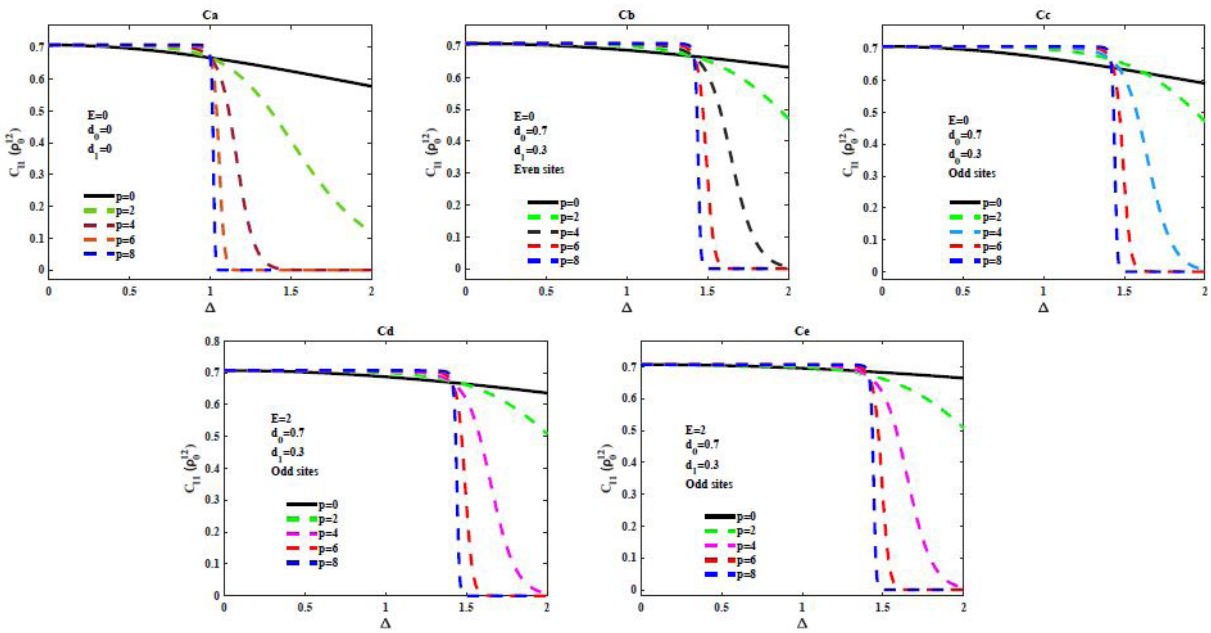

Figure 1: Bipartite quantum coherence $C_{l1}(\rho_0^{12})$ depending on the anisotropy parameter (with J = 1), for different iterations of QRG. For critical needs and compared to the case (**Cb,Cc,Cd**) where the effect of polarization (E) and/or modulated DM interaction have been taken into account; the graph (**Ca**) translates the behavior of the coherence of the system in the absence of environment and interactions.

by considering Eq. (18)) into Eq. (20)) as,

$$C_{l1}\left(\rho_0^{123}\right) = C_{l1}\left(\rho_0^{12}\right) + C_{l1}\left(\rho_0^{23}\right) + C_{l1}\left(\rho_0^{13}\right) = \\ = \frac{4\sqrt{(D(n)+\varepsilon)^2+1}}{q\left(|\alpha|^2+|\beta|^2+|\gamma|^2\right)} + \frac{2\left((D(n)+\varepsilon)^2+1\right)}{q(q+\Delta)\left(|\alpha|^2+|\beta|^2+|\gamma|^2\right)} \tag{25}$$

Such as it has been demonstrated in [21] , it has also been found here that the nearest neighbors are strongly correlated than the next nearest neighbors in a block. This correlations strend is the expression of the position dependency of bipartite quantum coherence and entanglement within a block.

To further elucidate the effect of the staggered part of the modulated DM interaction and the Electric polarisation on quantum coherence for different number of spins, we plot the ℓ1-norm of the nearest and next-neighbors (see Fig. 4 and Fig. 5) for different

QRG iterations respectively in Fig. 1 to Fig. 2 and in (Fig. 4) to (Fig. 5). The number N of total spins in the system at p^{th} iteration is given by $N = {}^{3p+1}$. dC_{l1} $(\rho_0{}^{12})$ depending on the anisotropy , for different iterations of QRG (with J = 1). Compared to case (b), case (a) exhibits modulated behavior in the absence of the influence of DM interactions. At low temperature and in the limit of the small number of spins (i.e for the first two iterations of the QRG), the bipartite quantum coherence decreases monotonically with the anisotropy (Fig. 1) and adopts a non-analytical behavior when much more of iterations are considered.

The transition from analytical behavior to non-analytical behavior of coherence is a particular characteristic of entanglement [22] which also reflects a signature of quantum phase transitions occurring at a critical value of the anisotropy Δc = 1 (Fig. 1.(Ca)) [21]. The corresponding quantum transition point becomes dynamic according to the variation of DM interactions coefficient and the staggered part of DM. Increasing

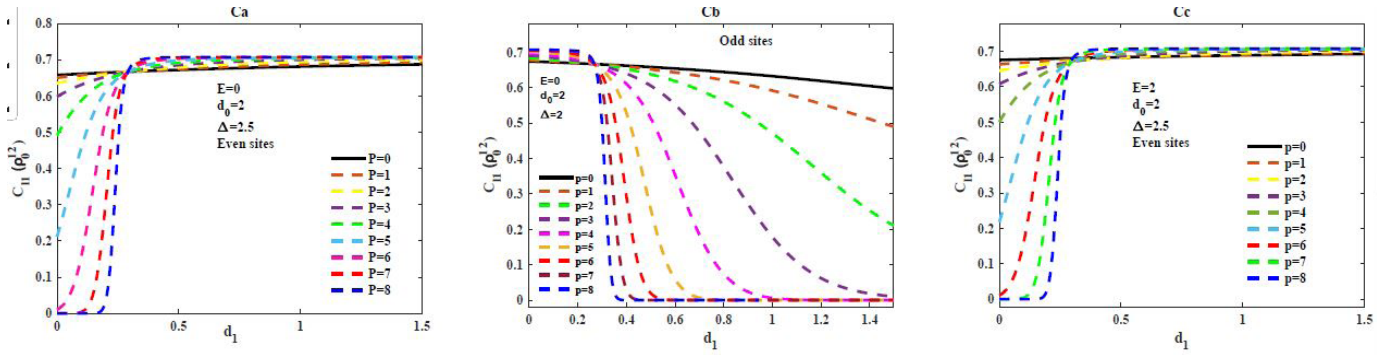

Figure 2: Bipartite quantum coherence C_{l1} $(\rho_0{}^{12})$ depending on the staggered part < d1 > of the DM interaction for different iterations of QRG (with J = 1).

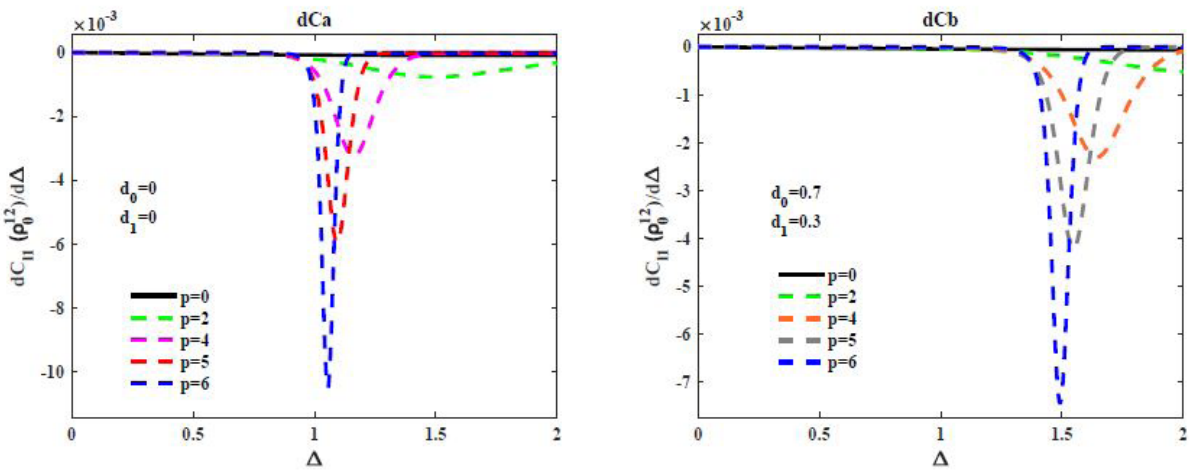

Figure 3; Influence of the modulated DM interaction on the dynamics of the first derivative bipartite quantum coherence.

the value of the staggered part of the DM parameter, shifts the QPT to the right in a coherent manner both in the case of even sites (Fig. 1.(Cb)) and for odd sites (Fig. 1.(Cc)) according to the law

$$\Delta_c = \sqrt{1 + (D(n) + \varepsilon)^2} \text{ [34].}$$

However, the black color curve obtained for odd sites (Fig. 1.(Cc)) in the absence of renormalization is shifted from the critical point, which reflects a delay in the entanglement which will be immediately corrected by the electric polarization for electric field values E ≥ 2 (Fig. 1.(Cd)). It emerges from this observation that the electric polarization reinforces the quantum correlations in the direction of displacement of the information.

In the thermodynamic limit (temperature close to absolute zero) and according to Bethe's Ansatz theory, Δc establishes the boundary between the phase of maximally correlated spins (liquid phase of spins) and the phase of completely uncorrelated spins (N´eel phase) [1]. The case of the even sites (Fig. 2.(Ca)) contrary to that of odd sites (Fig. 2.(Cb)) presents a liquid phase of predominant spins and whose correlation time will be more important depending on the value of the staggered parts d1 of the DM and d1 of the magnetic field. A phase alternation is observed when the dynamics of the chain depends on the site of occupation of the spin. The non-analytical nature of quantum coherence can be visualized from the divergence of its first derivative with respect to Δ as shown in (Fig. 3). The degree of non-analyticity can be estimated from the increasing depth of the minimum C and its variation as a function of the number of iterations. Each curve of the derivative reveals a minimum which becomes singular as one approaches the critical point. This singular behavior of quantum coherence is increasingly noticeable as the number of spins in the system increases.

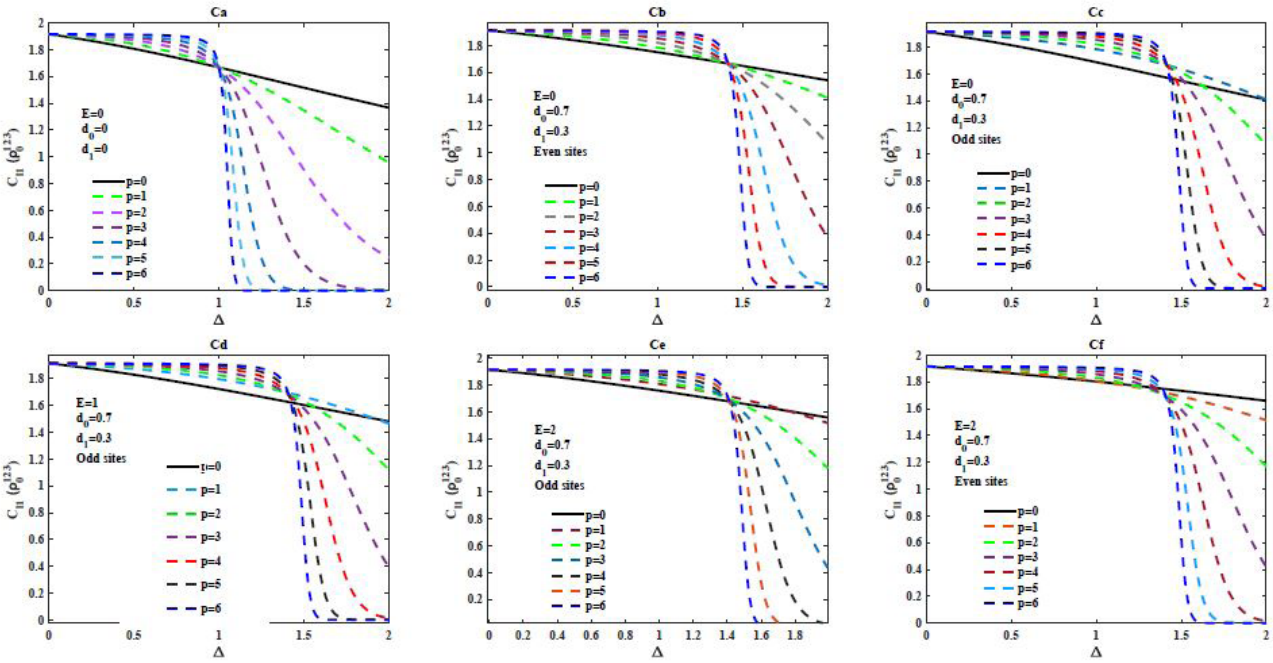

Figure 4: Tripartite quantum coherence Cl1 (ρ_0^{123}) depending on the anisotropy parameter for different iterations QRG (with J = 1). Compared to the case (Cb), the case (Ca) presently behaves in the absence of the influence of modulated DM interactions and the bias field. On the other hand, the cases (Cc, Cd, Ce, Cf) highlight the contribution of modulated DM interactions and the bias field. .

Dynamics of Quantum Correlations: Monogamy of Entanglemet

The monogamy of entanglement is based on the condition that if two parts of a multipartite system are maximally entangled then no entanglement can be transferred to the rest of the parts of the system. It has already been the subject of several researches as a parameter for measuring quantum correlations [35–37], through the following formula:

$$M = \sum_{n=2}^{N} C_{1:n} - C_{1:2\cdots N} \tag{26}$$

where C represents the concurrency (the measurement parameter of quantum entanglement). The bipartite concurrency of state with a single partition of the density matrix is a function of the eigenvalues of the transfer matrix ρ defined by:

$$C(\rho) = \max\left\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\right\} \tag{27}$$

where $\lambda 1 \geq \lambda 2 \geq \lambda 3 \geq \lambda 4$ are eigenvalues of the reduced density matrix R defined by:

$$R = \rho\left(\sigma^y \otimes \sigma^y\right)\rho^*\left(\sigma^y \otimes \sigma^y\right) \tag{28}$$

with ρ^* the complex conjugate of ρ . In the case of a pure state with N partitions, the concurrency is said to be multipartite and defined by:

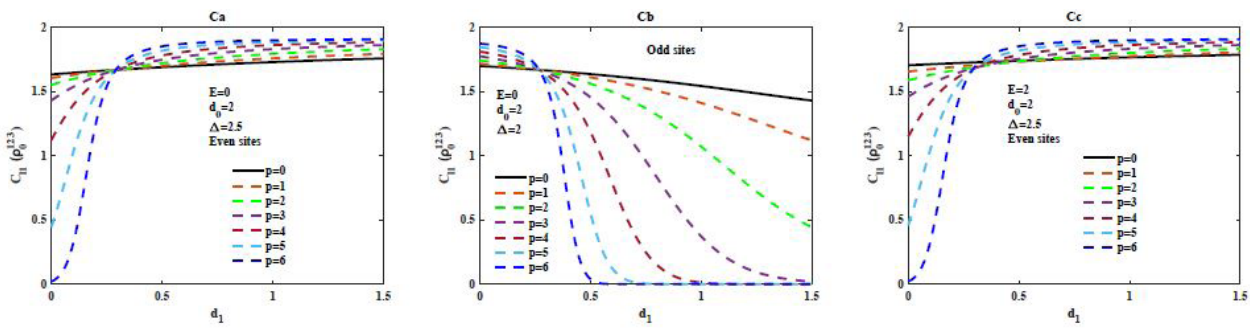

Figure 5: Tripartite quantum coherence Cl1 (ρ_0^{123}) depending on the staggered part d_1 for different iterations QRG (with $J = 1$). In addition to the contribution of the shifted part of the DM interaction, the influence of the polarization and the nature of the localization site (even or odd site) of the spin in the chain are highlighted.

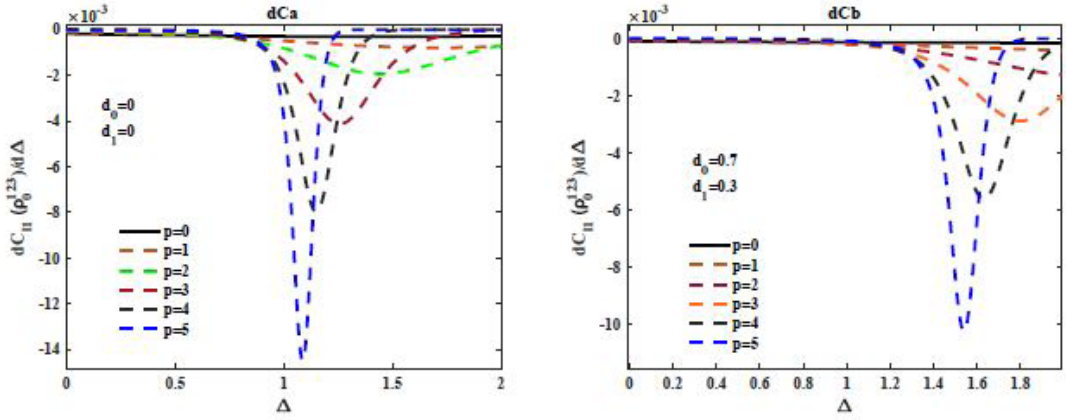

Figure 6: Influence of the modulated DM interaction on the dynamics of the first derivative of the tripartite quantum coherence $dC_{l1}(\rho_0^{123})$ depending on the anisotropy , for different iterations of QRG (with $J = 1$). Compared to the case (Cb), the case (Ca) presently modulated behavior in the absence of the influence of DM interactions.

concurrency is said to be multipartite and defined by:

$$C_d^N(\rho) = \sqrt{\frac{d}{2m(d-1)}\left(N - \sum_{i=1}^{N} Tr(\rho_i)^2\right)} \tag{29}$$

where $m = 2^{(N-1)} - 1$. represents the dimension of the Hilbert space of the system, ρi the partial density matrix obtained by the partial trace of the density matrix ρ . In application of the relation Eq. (29)) the expressions of the competitions obtained respectively between the 1st and the 2nd spin of a block, between the 1st and the 3rd spin of a block and between the 2nd and the 3rd spin of a block are identical to those of the coherences given by Eq. (23)) and Eq. (27)) i.e. $C_{l1}(\rho_0^{12}) = C_{l1}(\rho_0^{12}) = C_{l1}(\rho_0^{23}) = C_{l1}(\rho_0^{23})$ = and $C_{l1}(\rho_0^{13}) = Cl1(\rho_0^{13})$ for the bipartite cases. On the other hand, by making an extension of Eq. (28)) for the study of tripartite entanglement, it is found in this case that d = 8 and N = 3 then m = 3 ; leading to the following expression of concurrency:

$$C\left(\rho_0^{123}\right) = \sqrt{\frac{4}{21}\left(3 - \sum_{i=1}^{3} Tr(\rho_i)^2\right)} \tag{30}$$

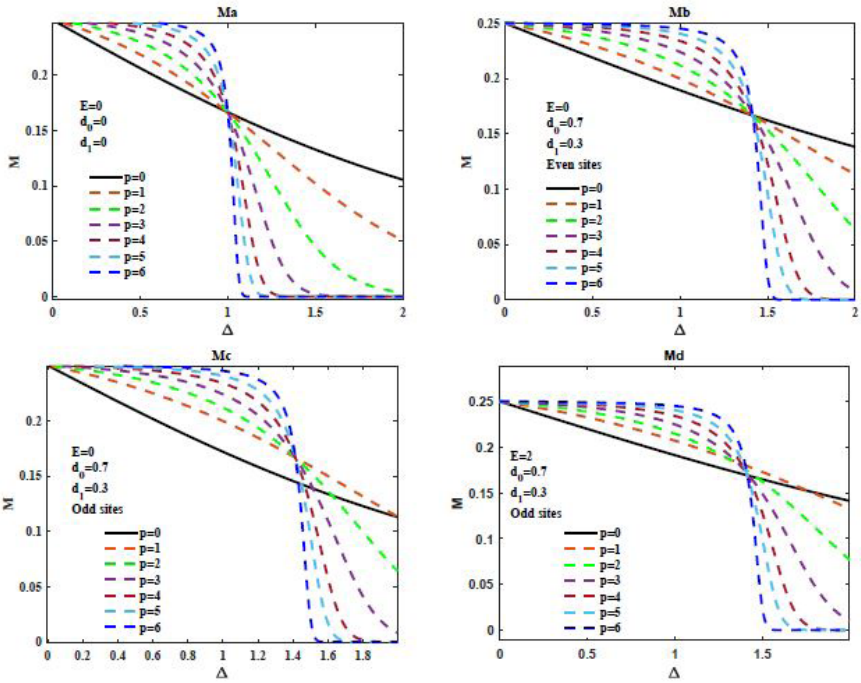

Figure 7: Monogamy M = $Cl1$ (ρ_0^{13}) as a function of the anisotropy parameter for different QRG iterations (with J = 1). Compared to the case (**Mb**) which takes into account the contribution of the DM interactions, the case (**Ma**) presents a behavior in the absence of the influence of the modulated DM interactions and of the polarization field. On the other hand, the cases (**Mc, Md**) highlight the contribution of modulated DM interactions and of the polarization field.

where the term $\Sigma^3_{i=1}$Tr(ρi)2 represents the sum of the traces of the squares of the reduced matrices defined by the relations Eq. (22)), hence the following relation of the tripartite entanglement reads:

$$C\left(\rho_0^{123}\right) = \frac{2}{\sqrt{21}}\sqrt{\left(C_{l1}\left(\rho^{12}\right)\right)^2 + \left(C_{l1}\left(\rho^{23}\right)\right)^2 + \left(C_{l1}\left(\rho^{13}\right)\right)^2} \tag{31}$$

From the definition of monogamy of entanglement

$$M = C_{l1}\left(\rho_0^{12}\right) + C_{l1}\left(\rho_0^{13}\right) + C_{l1}\left(\rho_0^{1,23}\right) \tag{32}$$

the following relation of the monogamy of entanglement can be obtained:

$$M = C_{l1}\left(\rho_0^{13}\right) = \frac{2\left|\alpha\right|\left|\gamma\right|}{\left|\alpha\right|^2 + \left|\beta\right|^2 + \left|\gamma\right|^2} \tag{33}$$

where C_{l1}($\rho_0^{1,23}$) is the l1-norm of the consistency between qubit 1 and the bipartite block{23} , with

$$\rho_0^{1,23} = \rho_0^1 \otimes \rho_0^{23} \tag{34}$$

the partial density matrix of 8×8 order. The monogamy of entanglement presents an analytical behavior which evolves towards a non-analytical behavior after a change of state around a value of the anisotropy C = 1 with the increase in the number of iterations of the QRG (Fig. 7(Ma)). This change of behavior reflects a quantum phase transition around the quantum transition point (QPT) defined by the critical value of the anisotropy $\Delta_C = 1$. The QPT moves with the variation of the critical value of the anisotropy due to the variation of the DM interaction coefficient by its staggered part d0 , as shown in (Fig. 7,(Mb)-(Md)). However, the monogamy of the entanglement displays a polygamous nature for the first three steps of the QRG and a monogamous nature after more iterations also depending on the value of the DM interaction.

In fact, for d0 ≥ 0 , the dynamics of QPT is such that the length and coherence time becomes large according to the value of DM interaction parameter. This coherence time will be longer stable with the increases of the value of the magnetic field intensity by its staggered part. The long coherence time (length) is the consequence of monogamous quantum nature which is very important in quantum information technology, for example in the field of telecommunications for designing the channels of information transport with less risk of loss of information over reasonable distances.

Conclusion

We have studied the dynamics of quantum coherence of a renormalized and polarized 1/2−XXZ Heisenberg spin chain by an electric field, in the presence of the modulated DM interaction and of the magnetic field. We used as a theoretical approach, the ℓ_1-norm method associated with the density matrix to measure the impact of the polarization factor, of the modulated part of DM interaction and of the magnetic field, on the dynamics of the quantum transition point of bipartite and tripartite states through the quantum coherence factor. The results obtained show how much the dynamics of the critical point, depends considerably on the parity of the system defined by the staggered part of the DM interaction (d_1) and of the applied magnetic field (h_1). This staggered part that helps to improve the value of the intensity of DM and of the magnetic field, generates equally a phase alternation between the liquid phase of the spins towards the Ne'el phase and vice versa. This setting help to stabilize the system in a phase and contributes not only to the variation of the quantum transition point, but even more to increase the entanglement delay in the systems. The polarization due to the electric field, reinforces the quantum correlations by an arrangement of the spins along the direction of the polarization wave vector, which is of very great interest in the control of the quantum coherence during the transport of the information by spin chains.

References

1. Sachdev S. Quantum phase transitions. Physics world. 1999 Apr 1;12(4):33.

2. okovi D, Osterloh A. On polynomial invariants of several qubits. Journal of mathematical physics. 2009 Mar 1;50(3).

3. Garbe L, Egusquiza IL, Solano E, Ciuti C, Coudreau T, Milman P, Felicetti S. Superradiant phase transition in the ultrastrong-coupling regime of the two-photon Dicke model. Physical Review A. 2017 May 23;95(5):053854.

4. Garbe L, Bina M, Keller A, Paris MG, Felicetti S. Critical quantum metrology with a finite-component quantum phase transition. Physical review letters. 2020 Mar 27;124(12):120504.

5. Hnati M, Honkonen J, Luivjansk T. Symmetry breaking in stochastic dynamics and turbulence. Symmetry. 2019 Sep 23;11(10):1193.

6. Friedman Z. Critical exponents for the three-dimensional ising model from the realspace renormalization group in two dimensions. Physical Review Letters. 1976 May 31;36(22):1326.

7. Joya W, Khan S, Khalid Khan M, Alam S. Analytic renormalized bipartite and tripartite quantum discords with quantum phase transition in XXZ spins chain. The European Physical Journal Plus. 2017 May;132:1-2.

8. Baumgratz T, Datta A. Quantum enhanced estimation of a multidimensional field. Physical review letters. 2016 Jan 21;116(3):030801.

9. Shao LH, Xi Z, Fan H, Li Y. Fidelity and trace-norm distances for quantifying coherence. Physical Review A. 2015 Apr 15;91(4):042120.

10. Boissonneault M, Gambetta JM, Blais A. Improved qubit bifurcation readout in the straddling regime of circuit QED. Physical Review A. 2012 Aug 23;86(2):022326.

11. Carmele A, Reitzenstein S. Non-Markovian features in semiconductor quantum optics: quantifying the role of phonons in experiment and theory. Nanophotonics. 2019 Apr 23;8(5):655-83.

12. Nakahara M, Sasaki Y, editors. Quantum Information and Quantum Computing- Proceedings Of Symposium. World Scientific; 2012 Sep 17.

13. Batra A, Kladnik G, Vzquez H, Meisner JS, Floreano L, Nuckolls C, Cvetko D, Morgante A, Venkataraman L. Quantifying through-space charge transfer dynamics in -coupled molecular systems. Nature communications. 2012 Jan;3(1):1086.

14. Haddadi S, Ghominejad M, Akhound A, Pourkarimi MR. Entropic uncertainty relation and quantum coherence under Ising model with DzyaloshinskiiMoriya interaction. Laser Physics Letters. 2021 Jul 9;18(8):085204.

15. Haddadi S, Pourkarimi MR, Wang D. Tripartite entropic uncertainty in an open system under classical environmental noise. JOSA B. 2021 Sep 1;38(9):2620-7.

16. Baumgratz T, Cramer M, Plenio MB. Quantifying coherence. Physical review letters. 2014 Sep 29;113(14):140401.

17. Yao Y, Xiao X, Ge L, Sun CP. Quantum coherence in multipartite systems. Physical Review A. 2015 Aug 12;92(2):022112.

18.Balazadeh L, Najarbashi G, Tavana A. Quantum renormalization of spin squeezing in spin chains. Scientific Reports. 2018 Dec 12;8(1):17789.

19.Khan S, Khan K. Tripartite quantum correlations in the renormalized space of Heisenberg Ising spin1/2 chain. Physica B: Condensed Matter. 2018 Sep 15;545:289-96.

20. Ma FW, Liu SX, Kong XM. Quantum entanglement and quantum phase transition in the X Y model with staggered Dzyaloshinskii-Moriya interaction. Physical Review A. 2011 Oct 3;84(4):042302.

21. JoyiaW, Khan S, Khan K. Bipartite and tripartite quantum coherence with entanglement in XXZ Heisenberg spins-1/2 chain under renormalization group method. Physica B: Condensed Matter. 2021 Jan 15;601:412663.

22. Jafari R, Kargarian M, Langari A, Siahatgar M. Phase diagram and entanglement of the Ising model with Dzyaloshinskii-Moriya interaction. Physical Review B. 2008 Dec 10;78(21):214414.

23. E.Mehran, S. Mahdavifar, and R. Jafari., Physical Review A 89 (4), 094434(2014).

24. Gu FL, Liu J, Mei F, Jia S, Zhang DW, Xue ZY. Synthetic spinorbit coupling and topological polaritons in JaneysCummings lattices. npj Quantum Information. 2019 Apr 30;5(1):36.

25. Hur N, Park S, Sharma PA, Ahn JS, Guha S, Cheong SW. Electric polarization reversal and memory in a multiferroic material induced by magnetic fields. Nature. 2004 May 27;429(6990):392-5.

26. Katsura H, Nagaosa N, Balatsky AV. Spin current and magnetoelectric effect in noncollinear magnets. Physical review letters. 2005 Jul 28;95(5):057205.

27. Fouokeng GC, Fodouop FK, Tchoffo M, Fai LC, Randrianantoandro N. Metamagnetoelectric effect in

multiferroics. Journal of Magnetism and Magnetic Materials. 2018 May 1;453:118-24.

28. Soumyanarayanan A, Reyren N, Fert A, Panagopoulos C. Emergent phenomena induced by spinorbit coupling at surfaces and interfaces. Nature. 2016 Nov 24;539(7630):509-17.

29. Cui Q, Liang J, Shao Z, Cui P, Yang H. Strain-tunable ferromagnetism and chiral spin textures in two-dimensional Janus chromium dichalcogenides. Physical Review B. 2020 Sep 22;102(9):094425.

30. Ga Y, Cui Q, Zhu Y, Yu D, Wang L, Liang J, Yang H. Anisotropic Dzyaloshinskii-Moriya interaction protected by D 2d crystal symmetry in two-dimensional ternary compounds. npj Computational Materials. 2022 Jun 8;8(1):128

31. Cui Q, Wang L, Zhu Y, Liang J, Yang H. Magnetic anisotropy, exchange coupling and DzyaloshinskiiMoriya interaction of two-dimensional magnets. Frontiers of Physics. 2023 Feb;18(1):13602.

32. Fumani FK, Beradze B, Nemati S, Mahdavifar S, Japaridze GI. Quantum correlations in the spin-1/2 Heisenberg XXZ chain with modulated Dzyaloshinskii-Moriya interaction. Journal of Magnetism and Magnetic Materials. 2021 Jan 15;518:167411.

33. Japaridze GI, Cheraghi H, Mahdavifar S. Magnetic phase diagram of a spin-1/2 X X Z chain with modulated Dzyaloshinskii-Moriya interaction. Physical Review E. 2021 Jul 26;104(1):014134.

34. Alcaraz FC, Wreszinski WF. The heisenberg xxz hamiltonian with dzyaloshinsky-moriya interactions. Journal of statistical physics. 1990 Jan;58:45-56.

35. Coffman V, Kundu J, Wootters WK. Distributed entanglement. Physical Review A. 2000 Apr 10;61(5):052306.

36. Giorgi GL. Monogamy properties of quantum and classical correlations. Physical Review A. 2011 Nov 2;84(5):054301.

37. Galve F, Giorgi GL, Zambrini R. Orthogonal measurements are almost sufficient for quantum discord of two qubits. Europhysics Letters. 2011 Nov 9;96(4): 40005.